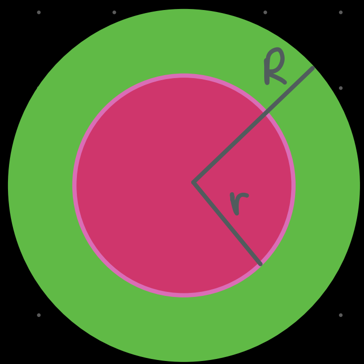
A solid sphere of radius R is made of an insulating material. It holds a charge Q which is distributed evenly throughout the sphere and gives it a uniform volume charge density ρ.



a) How much charge is enclosed by a concentric Gaussian sphere of radius r where $0 < r < R$?

We know volume charge density $\rho = \dfrac{\text{charge}}{\text{volume}}$

where charge = Q and volume of a sphere is $\frac{4}{3}\pi r^3$.

$\rho = \dfrac{Q}{V} \longrightarrow Q = \rho V = \rho\left(\frac{4}{3}\pi r^3\right)$ where r is the radius of the Gaussian surface and not R - the radius of the charged sphere. We only care about the amount of charge enclosed by the (smaller) Gaussian sphere.

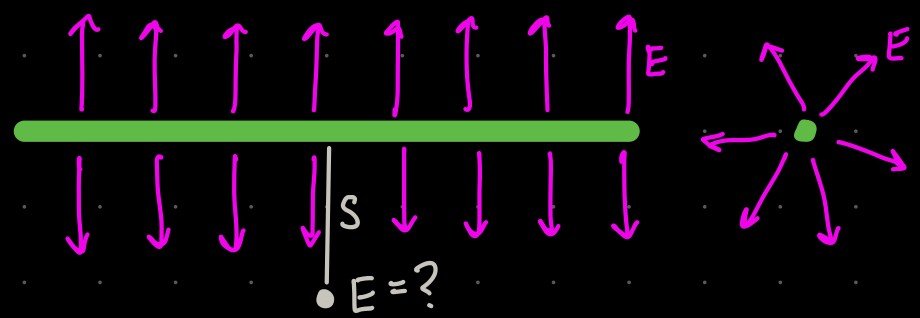
(b) What is the magnitude of the electric field produced by the charged sphere inside the sphere at a radial distance r?

Start with Gauss's Law: $\oint \vec{E} \cdot d\vec{A} = \dfrac{q_{enc}}{\epsilon_0}$

where $q_{enc} = \rho\left(\frac{4}{3}\pi r^3\right)$, found in part (a). Then we can simplify the integral: $\vec{E}$ is parallel to $d\vec{A}$ so $\vec{E} \cdot d\vec{A} = E\,dA$. E will have the same value everywhere on the Gaussian surface, so E = constant. $\oint \vec{E} \cdot d\vec{A} = \oint E\,dA = E \oint dA$ and $\oint dA = 4\pi r^2$ = surface area of sphere. Then put it all back into Gauss's Law:

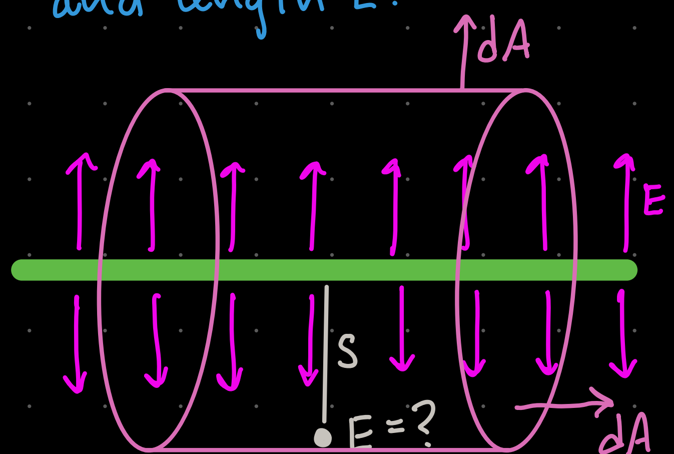
$$\oint \vec{E} \cdot dA = \frac{q_{enc}}{\epsilon_0} \longrightarrow E\left(4\pi r^2\right) = \frac{\rho\left(\frac{4}{3}\pi r^3\right)}{\epsilon_0} \longrightarrow E = \frac{\rho r}{3\epsilon_0}$$

A very long (nearly infinite) wire is made of an insulating material. The wire carries a charge Q which is distributed evenly along the entire length of the wire, giving it a uniform linear charge density λ. What is the magnitude of the electric field produced by the charged wire at a radial distance s from the wire?



To apply Gauss's Law we need to choose a Gaussian surface to enclose the wire.

We want to make the math easy by choosing a surface that will have $\vec{E} \parallel \vec{A}$ or $\vec{E} \perp \vec{A}$. A cylinder will do this. Enclose the wire with a Gaussian cylinder of radius s and length L.



$d\vec{A}$ and $\vec{E}$ are parallel on the rounded surface and $d\vec{A}$ and $\vec{E}$ are perpendicular on the circular ends.

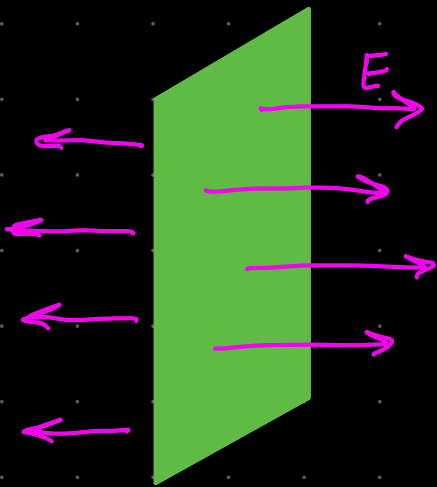
$$\oint \vec{E} \cdot d\vec{A} = \oint E\,dA + \oint 0$$

E will be the same value everywhere on the rounded surface, so E = constant $\therefore \oint E\,dA = E \oint dA$ and the surface area of a cylinder is $\oint dA = 2\pi s L$

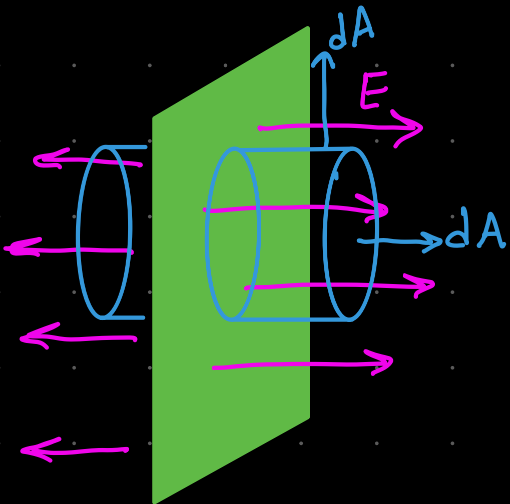
linear charge density $\lambda = Q/L \rightarrow Q = \lambda L = q_{enc}$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0} \longrightarrow E(2\pi s L) = \frac{\lambda L}{\epsilon_0} \longrightarrow E = \frac{\lambda}{2\pi s \epsilon_0}$$

Given an infinite sheet of charge, which Gaussian surface is best suited to calculate E near the charged sheet?

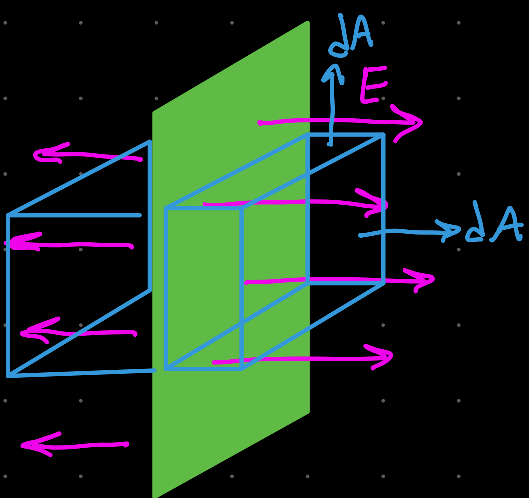


Near the surface of the charged sheet, the E-field is perpendicular to the sheet. To make it easier to evaluate the integral $\oint \vec{E} \cdot d\vec{A}$, we should choose a surface such that $\vec{E}$ and $d\vec{A}$ are either parallel or perpendicular to each other.



This cylinder is a good choice because $\vec{E}$ is parallel to dA on the two ends of the cylinder and $\vec{E}$ is perpendicular to dA on the rounded surface.

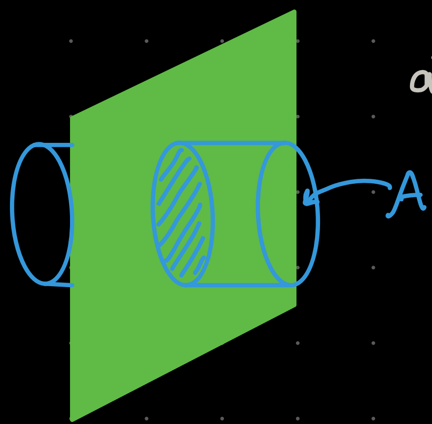
$$\oint \vec{E} \cdot d\vec{A} = \underbrace{\oint E\,dA + \oint E\,dA}_{\text{2 ends}} + \underbrace{0}_{\text{round side}}$$



This cube is also a good choice, for similar reasons we used with the cylinder.

$$\oint \vec{E} \cdot d\vec{A} = \underbrace{\oint E\,dA + \oint E\,dA}_{\text{front and back}} + \underbrace{0}_{\text{sides}}$$

A very large plane sheet is made of an insulating material. The sheet carries a charge Q, which is distributed evenly over the entire surface area, giving it a uniform surface charge density σ. A Gaussian cylinder of length L and cross-sectional area A encloses a portion of the charged sheet.



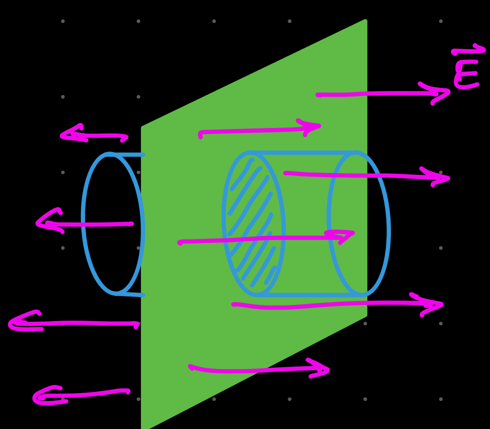
a) What is the charge enclosed by the Gaussian cylinder?

The section of the sheet enclosed by the cylinder has an area $= A$, the same cross-sectional area of the cylinder.

Surface charge density : $\sigma = \dfrac{Q}{A}$

so $Q_{enc} = \sigma A$

b) What is the magnitude of the $\vec{E}$-field produced by the surface charge at a distance s from the sheet?



For an infinite sheet, $\vec{E}$ is perpendicular to the sheet. This means $\vec{E}$ is uniform and constant, no matter where you measure $\vec{E}$. So the distance from the sheet doesn't matter.

$$\oint \vec{E} \cdot d\vec{A} = \underbrace{\int E\,dA + \int E\,dA}_{\text{2 ends of cylinder}} = 2E\int dA = 2EA$$

$$\oint \vec{E} \cdot dA = \frac{Q_{enc}}{\epsilon_0}$$ where $Q_{enc} = \sigma A$

$$2EA = \frac{\sigma A}{\epsilon_0}$$ A will cancel

$$E = \frac{\sigma}{2\epsilon_0}$$ no dependance on distance from sheet, as predicted