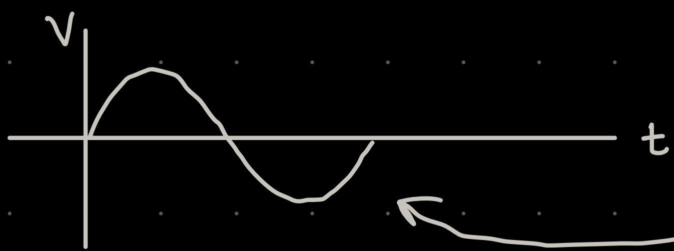
Practice 6.1.1

An alternating current (AC) source supplies a sinusoidally varying voltage that can be described with the function $V(t) = V_0 \sin(2\pi f t)$, where V_0 is the maximum voltage, f is the frequency, and t is the time. If the oscillation frequency of this source is 60 Hz, what is the oscillation period?

The source voltage varies in time: $V(t) = V_0 \sin(2\pi f t)$.
We're told the frequency is 60 Hz, or 60 cycles per sec.
One cycle is one full sine curve:



So 60 Hz means the voltage changes like this 60 times every second.
The period (T) is the amount of time it takes for the voltage to complete one cycle. The relationship between frequency and period is an inverse one:

$$T = \frac{1}{f}$$

Therefore, the oscillation period is $T = \frac{1}{60\,\text{Hz}} = \frac{1}{60}\,\text{s}$

Quick check on units: $\text{Hz} = \text{cycles/s}$ or $1/\text{s}$ or s^{-1}

So $\frac{1}{\text{Hz}} = \frac{1}{1/\text{s}} = \frac{1}{\text{s}^{-1}} = \text{s}$ ✓

Period is a measure of time so it should have units of seconds.

Practice 6.1.2

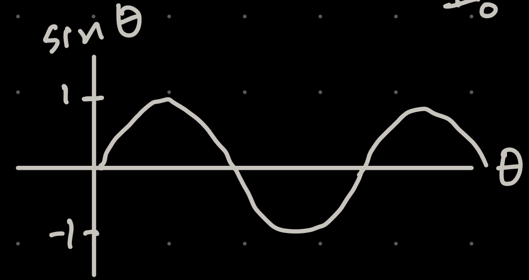
A resistor is connected in series with an AC source that provides a sinusoidal voltage of $V(t) = V_0 \sin(2\pi ft)$, where V_0 is the maximum voltage, f is the frequency, and t is the time. The current supplied by this source that flows through this resistor is described with the function $I(t) = I_0 \sin(2\pi ft)$, where I_0 is the maximum current.

What is the average power supplied by this AC source?

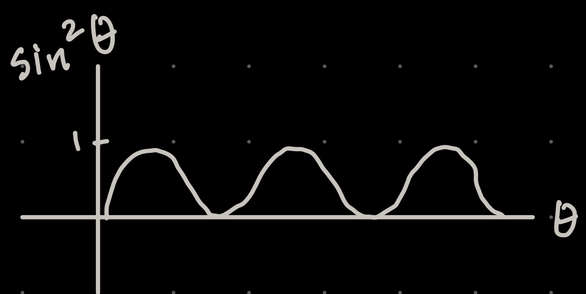
Again, the source voltage changes in time: $V(t) = V_0 \sin(2\pi ft)$
Now we're given information about how the current changes in time, too: $I(t) = I_0 \sin(2\pi ft)$.

We need to find the average power supplied by the AC source. Power is current × voltage:

$$P = IV = I_0 \sin(2\pi ft)\, V_0 \sin(2\pi ft)$$
$$= I_0 V_0 \sin^2(2\pi ft)$$



The average value of $\sin\theta$ is always zero. But $\sin^2\theta$ is different.



The max value of $\sin^2\theta$ is 1 and the min value is zero. So the average of $\sin^2\theta$ is always $\frac{1}{2}$

Average Power $= P_{av} = I_0 V_0 \left(\frac{1}{2}\right) = \frac{1}{2} I_0 V_0$

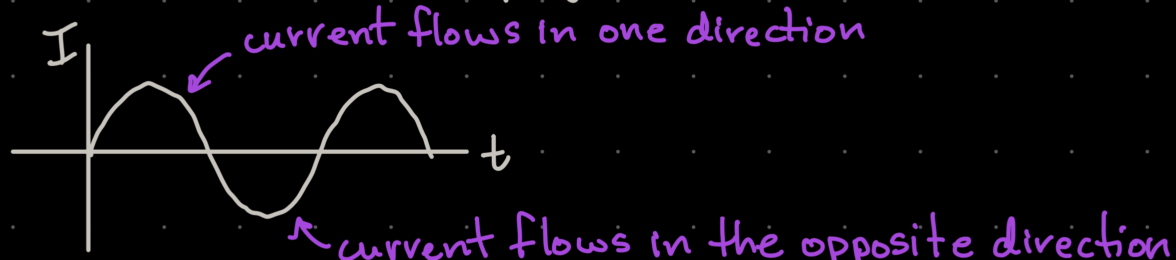
Practice 6.1.3

A resistor is connected in series with an AC source that provides a sinusoidal voltage of $V(t) = V_0 \sin(2\pi f t)$, where V_0 is the maximum voltage, f is the frequency, and t is the time. The current supplied by this source that flows through this resistor is described with the function $I(t) = I_0 \sin(2\pi f t)$, where I_0 is the maximum current.

What is the *rms* current in the resistor?

Given $V(t) = V_0 \sin(2\pi f t)$ and $I(t) = I_0 \sin(2\pi f t)$

We need to find the rms current, which is similar to an average value, but not exactly the same thing. We saw in the previous example how the average value of $\sin\theta$ is always zero. This is true mathematically but it doesn't make physical sense.



It doesn't make sense to say the average current in the resistor is zero since there is non-zero current flowing through the resistor. To account for this, we use rms instead of average.

rms = root-mean-square but we do the computation in reverse order. First, square the function to get rid of negative values, then take the mean (average), then take the square-root to get back to the dimensions we started with.

$I(t) = I_0 \sin(2\pi f t)$

square: $I_0^2 \sin^2(2\pi f t)$

mean: $I_0^2 \left(\frac{1}{2}\right)$ we know the avg of $\sin^2\theta$ is always $\frac{1}{2}$

root: $\frac{I_0}{\sqrt{2}}$

So the rms current is: $I_{rms} = \frac{I_0}{\sqrt{2}}$

This is a constant value (not changing in time) and we can use it in Ohm's law like we do with DC circuits.

Practice 6.1.4

A resistor is connected in series with an AC source that provides a sinusoidal voltage of $V(t) = V_0 \sin(2\pi f t)$, where V_0 is the maximum voltage, f is the frequency, and t is the time. The current supplied by this source that flows through this resistor is described with the function $I(t) = I_0 \sin(2\pi f t)$, where I_0 is the maximum current.

What is the *rms* voltage across the resistor?

Finding V_{rms} is the same process as finding I_{rms}. We will get the same result:

$$V_{rms} = \frac{V_0}{\sqrt{2}}$$

We can also use V_{rms} in Ohm's law just like in DC circuits.