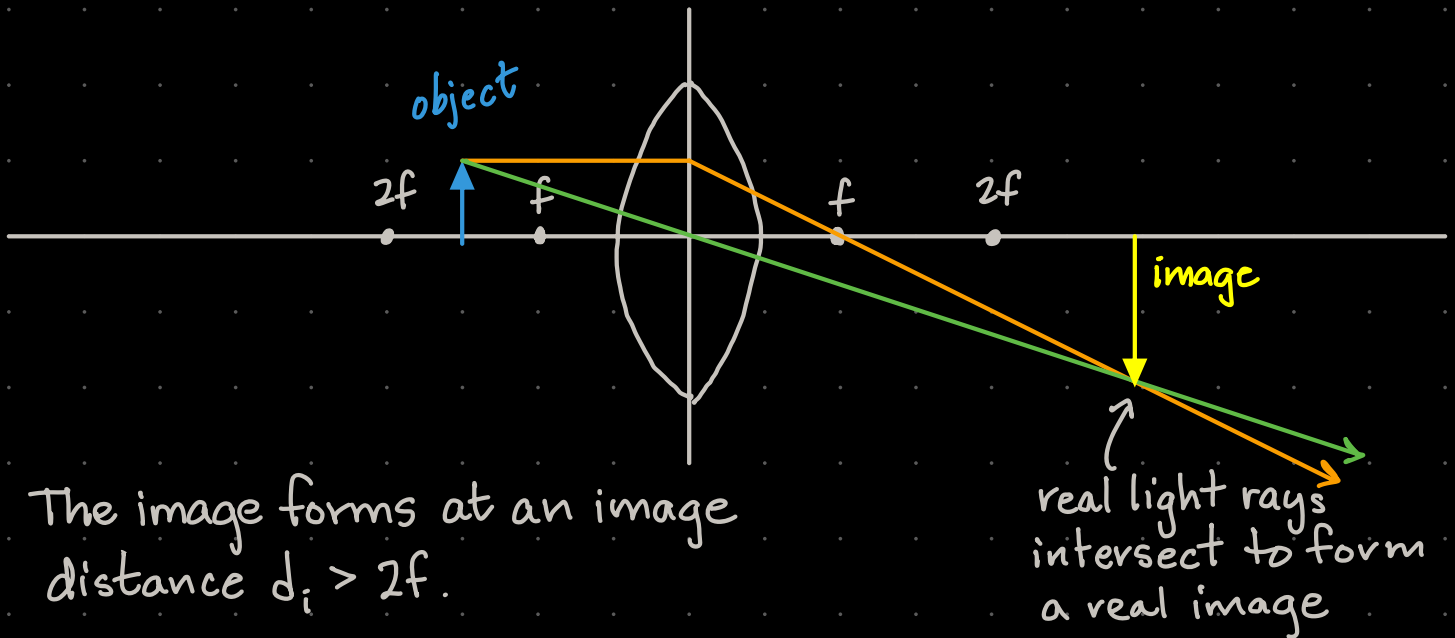


A convex lens has a focal length f . An object is placed at a distance between f and $2f$ on a line perpendicular to the center of the lens. The image formed is located at what distance from the lens?

A convex lens is a converging lens. When an object is located between f and $2f$, a real image will form on the far side of the lens.



The image forms at an image distance $d_i > 2f$.

We can use the lens equation to confirm this result.

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \rightarrow \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$$

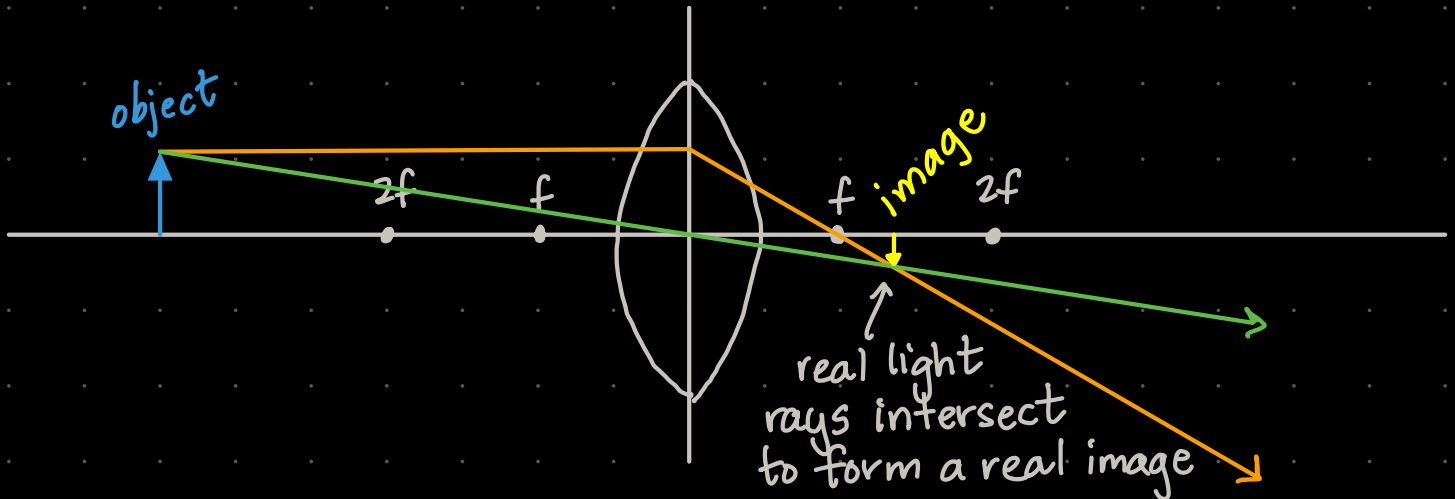
$$\text{If } d_o = f, \text{ then } \frac{1}{d_i} = \frac{1}{f} - \frac{1}{f} = 0 \rightarrow d_i = \frac{1}{0} = \infty \text{ no image forms}$$

$$\text{If } d_o = 2f, \text{ then } \frac{1}{d_i} = \frac{1}{f} - \frac{1}{2f} = \frac{2}{2f} - \frac{1}{2f} = \frac{1}{2f} \rightarrow d_i = 2f$$

This means for $f < d_o < 2f$, the image distance will be between $2f$ and ∞ . This supports our first result.

A convex lens has a focal length f . An object is placed between infinity and $2f$ from the lens along a line perpendicular to the center of the lens. The image is located at what distance from the lens?

When the object is located between ∞ and $2f$, the image forms between f and $2f$.



Using the lens equation to confirm:

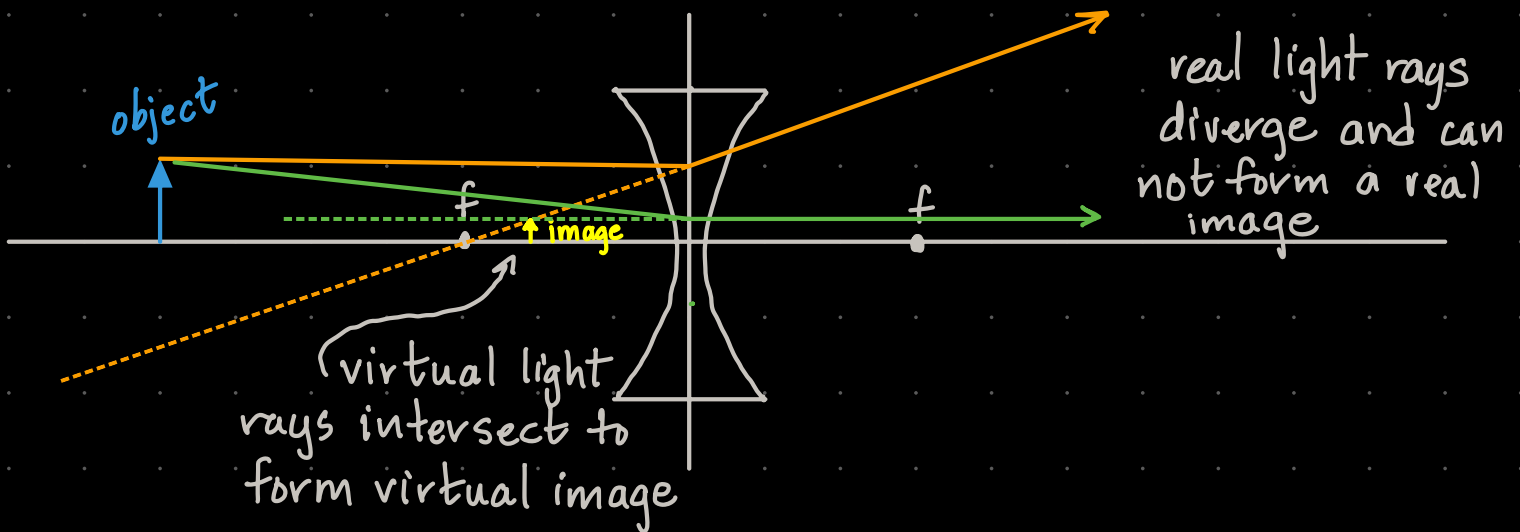
$$\text{If } d_o = \infty : \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \rightarrow \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{f} - \frac{1}{\infty} = \frac{1}{f}$$
$$d_i = f$$

$$\text{If } d_o = 2f : \frac{1}{d_i} = \frac{1}{f} - \frac{1}{2f} = \frac{2}{2f} - \frac{1}{2f} = \frac{1}{2f} \rightarrow d_i = 2f$$

This confirms that for $2f < d_o < \infty$, the image distance will be between f and $2f$.

An object is placed at 29.0 cm to the right of a diverging lens whose focal length is -25 cm.
What is the location of the image relative to the lens?

A concave lens is a diverging lens. A diverging lens always produces virtual images.



Now let's quantify this by using the lens equation.

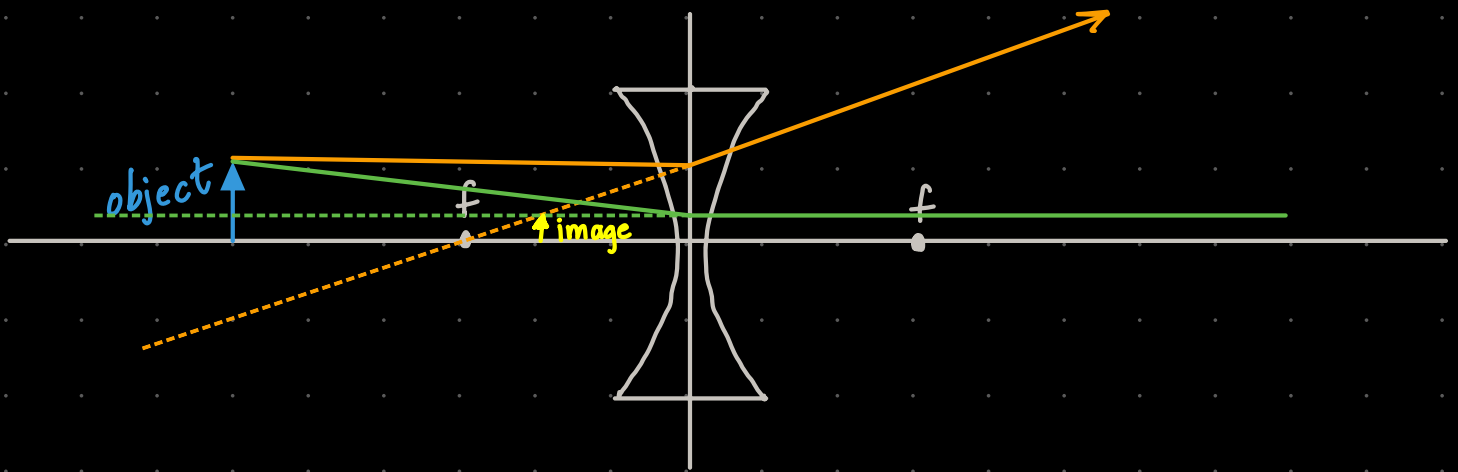
Given object distance: $d_o = 29$ cm and focal length $f = -25$ cm (negative because it's a diverging lens).

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \rightarrow \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{-25} - \frac{1}{29} = -0.074 \text{ cm}^{-1}$$

$$d_i = -13.4 \text{ cm}$$

measured on the same side of the lens as the object, confirming it is a virtual image.

A thin diverging lens has focal length $f = -12$ cm. If an object 9 cm tall is placed 24 cm from the lens, what is the height of the image?



Given: diverging lens, $f = -12$ cm
 object height, $h_o = 9$ cm
 object distance, $d_o = 24$ cm

Need to find magnification since $h_i = m d_o$ and magnification is $m = -\frac{d_i}{d_o}$. So we need to find the image distance.

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \rightarrow \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{-12} - \frac{1}{24} = \frac{2}{-24} - \frac{1}{24} = -\frac{3}{24}$$

$$d_i = -\frac{24 \text{ cm}}{3} = -8 \text{ cm}$$

Now the magnification:

$$m = -\frac{d_i}{d_o} = -\frac{(-8 \text{ cm})}{24 \text{ cm}} = +0.33 \quad \text{image is upright}$$

$$h_i = m h_o = (0.33)(9 \text{ cm}) = 3 \text{ cm}$$

A convex, or converging, lens with a focal length of 34 cm is used to observe an image of an apple that is 80 cm away from the lens. At what distance from the lens does the image form and what type of image is it?

Converging lens with $f = +34 \text{ cm}$, object distance $d_o = 80 \text{ cm}$.

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \rightarrow \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{34} - \frac{1}{80} = 0.0169 \text{ cm}^{-1}$$

$$d_i = +59 \text{ cm}$$

The image forms at 59 cm from the lens. Because the image distance is positive, the image is a real image and forms on the far side of the lens (opposite side to where the object is located).

A person's eye has a diameter of 2.5 cm and a converging lens that produces a clear, real image on the retina. When viewing an apple, the eye's lens has a focal length of 2.4 cm. What is the distance between the apple and the person's eyes?

Converging lens, $f = 2.4 \text{ cm}$

eye diameter = distance from lens to retina = image distance

$$d_i = 2.5 \text{ cm}$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \rightarrow \frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i} = \frac{1}{2.4} - \frac{1}{2.5} = 0.0167 \text{ cm}^{-1}$$

$d_o = 60 \text{ cm} \rightarrow$ The apple (object) is located 60 cm from the eye.

A nearsighted person wants to see an apple that is 7 meters away but can only clearly see objects that are at most 62 cm away from her eyes. Eyeglasses made of diverging corrective lenses can help her to see the apple clearly. If her glasses are 2.5 cm in front of her eyes, what should be the power of these corrective lenses to allow her to see the apple?

A nearsighted person can only focus on objects closer than her **far point** - the maximum distance at which she can see clearly. Everything beyond that appears blurry. The job of the corrective lens is not to magnify the apple, but to create a virtual image of the apple at a distance she can see clearly. A diverging lens produces an image closer to the eye than the actual object, placing it within the range where the nearsighted person can focus.

The lens must take the apple at 7 meters away and produce a virtual image of it no farther than her **far point** from her eye.

The glasses sit 2.5 cm in front of her eyes, so all distances must be measured from the lens, not from her eyes.

$$\begin{aligned}\text{Object distance (apple to lens): } d_o &= 7\text{ m} - 0.025\text{ m} \\ d_o &= 6.975\text{ m}\end{aligned}$$

The far point of her eye is 62 cm from her eye, which is $0.62\text{ m} - 0.025\text{ m} = 0.595\text{ m}$ from the lens.

The lens must form the virtual image no farther than 0.595 m in front of the lens (on the same side as the object). For a diverging lens, a virtual image on the same side of the lens as the object means the image distance is negative: $d_i = -0.595 \text{ m}$

Applying the lens equation:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{6.975} + \frac{1}{-0.595} = -1.537 \text{ m}^{-1}$$

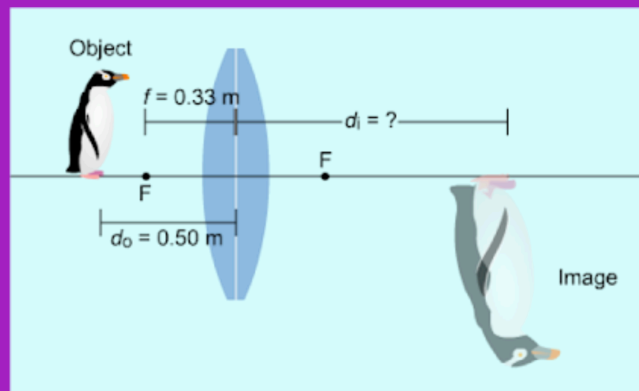
Converting to power:

The power of a lens in diopters is simply the inverse of the focal length in meters.

$$\text{Power} = \frac{1}{f} = -1.54 \text{ D}$$

The negative power confirms this is a diverging lens, as expected for nearsightedness. If you look at a prescription for eyeglasses for a nearsighted person, you will find that the optical power is negative and given in units of diopters. The lens spreads the light rays from the distant apple outward just enough that they appear to the eye to be coming from a point within her 62 cm range, allowing her to focus on them clearly.

Given the object distance and focal length of the converging lens in the figure, determine the image distance.



We're given $d_o = 0.5\text{ m}$ and $f = 0.33\text{ m}$. We need to find d_i .

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \rightarrow \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{0.33} - \frac{1}{0.5} = 1.03\text{ m}^{-1}$$

$$d_i = 0.97\text{ m}$$

Suppose the image of an object is upright and magnified 1.95 times when the object is placed 17.5 cm from a particular converging lens.

(a) Find the location of the image.

(b) Find the focal length of the lens.

Given: upright image (m is positive)

$$m = 1.95$$

object distance: $d_o = 17.5$ cm

converging lens (f is positive)

(a) To find the image distance without knowing the focal length, we can use the magnification.

$$m = \frac{-d_i}{d_o} \rightarrow d_i = -m d_o = -(1.95)(17.5 \text{ cm})$$

$$d_i = -34.1 \text{ cm} \quad (\text{negative} \rightarrow \text{virtual image})$$

(b) Now we know both d_o and d_i , so we can use the lens equation to find the focal length.

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{17.5} + \frac{1}{-34.1} = 0.0278 \text{ cm}^{-1}$$

$$f = 35.9 \text{ cm} \quad (\text{positive value} \rightarrow \text{converging lens})$$