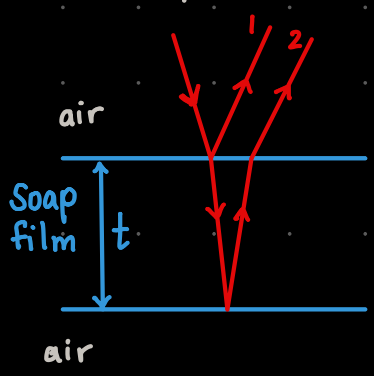
Light of frequency 6.00 × 10^{14} Hz illuminates a soap film (n = 1.33) having air on both sides of it. When viewing the film by reflected light, what is the minimum thickness of the film that will give an interference maximum when the light is incident normally on it?

We want to find the film thickness (t) that will cause reflected light to interfere constructively. We're given the frequency of the light but we need the wavelength. The speed of any wave is $v_{wave} = \lambda f$ and the speed of this wave is c - the speed of light. So: $c = \lambda f \longrightarrow \lambda = \frac{c}{f} = \frac{3 \times 10^8 \text{ m/s}}{6 \times 10^{14} \text{ Hz}} = 5 \times 10^{-7} \text{ m} = 500 \text{ nm}$

The wavelength of this light in air is $\lambda_o = 500$ nm. When this light travels in the soap film, the wavelength decreases: $\lambda_{film} = \frac{\lambda_o}{n_{film}} = \frac{500 \text{ nm}}{1.33} = 376$ nm.



Light ray 1 reflects at the air-soap interface. Since the light is reflecting from a higher refractive index medium, the reflected light is phase-shifted by 180°, or $\frac{1}{2}\lambda$. Light ray 2 transmits into the soap film, reflects from the soap-air interface, to then travel out of the film to then interfere with light ray 1. Light ray 2 is not phase shifted because it reflects from a lower refractive index medium.

Light ray 2 travels a longer distance than light ray 1 so there is a path difference. Ray 2 travels through the film twice, once going in and once coming out. Ray 2 travels $t + t = 2t$ farther than Ray 1. This means the path difference is $2t$.

For the two rays to interfere constructively, the rays must be in phase when they leave the film. Usually, if the path difference is equal to the wavelength (or an integer multiple of wavelengths: $m\lambda$), but this time we also need to consider how Ray 1 experiences a 180° phase shift which moves Ray 1 out of phase (by $\frac{1}{2}\lambda$) with Ray 2. In order for these rays to constructively interfere, we need the path difference to be half a wavelength (or integer multiples: $(m+\frac{1}{2})\lambda$). This is only because one light ray is phase-shifted. If both rays were phase-shifted, then the path difference needed for constructive interference would be $m\lambda$.

OK, now we are ready to set up the calculation:

path difference: $2t$

one phase shift, $2t = (m+\frac{1}{2})\lambda$ for constructive interference

λ_n = wavelength of light when travelling in the film

$$2t = (m+\tfrac{1}{2})\frac{\lambda_0}{n}$$ where $n = 1.33$ (index of soap film)

Solve for thickness: $$t = \frac{(m+\frac{1}{2})\lambda_0}{2n}$$

minimum thickness: $m = 0$

thicker films ($m = 1, 2, \ldots$) will also result in constructive interference.

minimum thickness: $$t = \frac{(0+\frac{1}{2})(500\text{ nm})}{2(1.33)} = 94\text{ nm}$$

Calculate the minimum thickness of a soap-bubble film ($n = 1.33$) that results in constructive interference in the reflected light if the film is illuminated with light whose wavelength in air is $\lambda = 460$ nm. What are the next two thinnest film thicknesses that will produce constructive interference?

This is very similar to the previous problem. It's a soap film surrounded by air on both sides of the film. The ray that reflects from the outer surface of the film is phase-shifted because it reflects from a higher n medium ($n_{soap} > n_{air}$) but the ray that travels into the film and reflects from the air at the inner surface of the film is not phase shifted ($n_{air} < n_{film}$).

The path difference is still $2t$ because Ray 2 travels into the film and back out again. For constructive interference, and with only one ray that is phase-shifted, we will set the path difference equal to $(m+\frac{1}{2})\lambda_n$.

$$2t = (m+\tfrac{1}{2})\lambda_n \longrightarrow 2t = (m+\tfrac{1}{2})\frac{\lambda_0}{n} \longrightarrow t = \frac{(m+\frac{1}{2})\lambda_0}{2n}$$

$m=0$: thinnest thickness to result in constructive interference

$m=1,2$: next two thinnest thicknesses

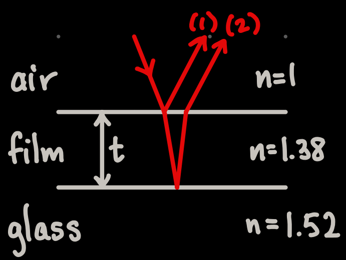
$$t = \frac{(0+\frac{1}{2})(460\,\text{nm})}{2(1.33)} = 86.5\text{ nm}$$

$$t = \frac{(1+\frac{1}{2})(460\,\text{nm})}{2(1.33)} = 259\text{ nm}$$

$$t = \frac{(2+\frac{1}{2})(460\,\text{nm})}{2(1.33)} = 432\text{ nm}$$

The antireflective glass shown is covered with a thin layer of magnesium fluoride (n = 1.38). The index of refraction of the glass is 1.52.

What minimum thickness of magnesium fluoride is needed to cause destructive interference of light whose wavelength is in the middle of the visible range at 555 nm?



This time we are looking for film thickness that will cause destructive interference of the reflected light. This way you will not see light reflecting from the glass (what we call 'glare').

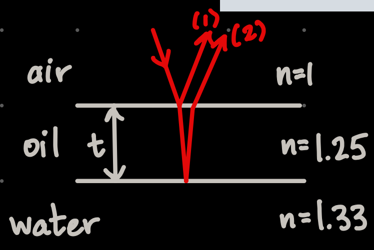
Ray 1 travels in air (n=1) and reflects from the film (n=1.38). Since this is a reflection from a higher index medium, the reflected Ray 1 will be phase shifted. Ray 2 travels in the film (n=1.38) and reflects from the glass (n=1.52), which is also a reflection from a higher n medium so the reflected Ray 2 is also phase shifted. Since both reflected rays are phase shifted by the same amount, they will remain in phase.

To ensure destructive interference, we will set the path diffence of 2t equal to $(m+\frac{1}{2})\lambda_n$ where m=0 for the minimum thickness.

$$2t = (m+\tfrac{1}{2})\lambda_n \longrightarrow 2t = (m+\tfrac{1}{2})\frac{\lambda_o}{n} \longrightarrow 2t = (0+\tfrac{1}{2})\frac{\lambda_o}{n} \longrightarrow t = \frac{\lambda_o}{4n}$$

minimum thickness for destructive interference : $t = \dfrac{555\,\text{nm}}{4(1.38)} = 100.5\ \text{nm}$

A thin layer of oil (n = 1.25) is floating on water (n = 1.33). What is the minimum nonzero thickness of the oil in the region that strongly reflects green light (λ = 530 nm)?



Similar to the previous problem, both reflected rays experience the same 180° phase shift because each one reflects from a higher n medium $n_{oil} > n_{air}$ for Ray 1 and $n_{water} > n_{oil}$ for Ray 2.

This time we're looking for the minimum film thickness that will cause constructive interference of the reflected rays. So we can set the path difference equal to the wavelength (m=1 for minimum thickness).

path difference : $2t = m\lambda_n$ m=1 for minimum thickness

$$2t = \lambda_n = \frac{\lambda_o}{n}$$

$$t = \frac{\lambda_o}{2n} = \frac{530\text{ nm}}{2(1.25)} = 212\text{ nm}$$